I present a trade model where the North and the South differ in their demand for quality and quality of intermediate (task) supply. The model explains some recent empirical findings that are not captured by existing international trade models: input trade liberalisation leads to quality upgrading of Southern exports, and Southern firms vary the quality of their products across destinations. The model suggests that an increase in the variety of imported intermediates from the North improves product quality and increases the probability of exporting. However, there exists industry heterogeneity in the effect of imported intermediates on the probability of exporting. (JEL F12, L11)

In 2004, Regal, a Turkish brand of consumer durables, launched a successful series of TV commercials. Each commercial advertised a particular product of Regal. The plot was the following. Inside an interrogation room, there is an interrogator and a customer. The interrogator offers the customer two brands of a product—a luxury brand and Regal. He explains to the customer that the Regal product shares the same features as the luxury one, but cheaper. Then, the interrogator asks her to choose between the two, and she invariably goes for the...
luxury one. The plot ends with the interrogator insulting the customer because of her *irrational* choice.

Regal is the low-end brand of Vestel Group which is one of the leading manufacturers of consumer durables in Turkey. The Group sells its products under different brands in the domestic, Middle-Eastern, and European markets. In each product line, the Group’s factory produces multiple brands that differ slightly in their appearance but share the same features. The brands, however, differ significantly from each other in terms of price.

There exists anecdotal evidence that Turkish exporters use inputs of differentiated quality to differentiate the quality of their products across markets. The anecdotal evidence comes from an interview that I conducted with an advisor to the Customs Administration of Turkey. Based on his extensive experience with Turkish exporters, he provided various examples of how Turkish manufacturers vary the quality of their inputs to produce different versions of the same product. Such strategies, he argued, are common among firms operating particularly in textiles, consumer durables and electronics. Using high-quality intermediates to produce high-quality products is a mechanism for quality-upgrading, and it is particularly relevant for Southern (developing) country firms since high-quality intermediates may not be available in their domestic markets.

In this paper, I develop an international trade model where a firm located in the South adapts its product quality to local demand—which I call quality-to-market—by changing the shares of low and high-quality intermediates.\(^1\) Assuming that Southern firms source high-quality inputs from the North (developed countries), the model introduces a new margin of trade adjustment: the extensive margin of imported inputs. It refers to the set of imported inputs at the firm-level, and its increase is associated with quality upgrading. The extensive margin of imports increases with specific trade costs on final goods, and it decreases with ad valorem trade costs on final goods and on intermediates. To study the decision to import, I embed the model in the heterogeneous firms trade model presented by Chaney (2008), who modifies the model developed by Melitz (2003). The extension suggests two important predictions. First, regardless of whether there is a fixed importing cost, there exists a cut-off productivity

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\(^1\)Görg, Halpern, and Muraközy (2010) also use the term "quality-to-market" when analysing empirically the strategy followed by Hungarian exporters. I discuss the findings of the paper below.
for Southern firms to import intermediates from the North. Second, input trade liberalisation is more likely to increase the probability of exporting in industries where input quality is an important determinant of product quality. Furthermore the extension neatly generates the Linder’s hypothesis (Linder 1961): countries are better at producing the products for which there is strong domestic demand. In particular, firms located in markets where consumer demand for quality is high are better at producing high-quality products.

With its focus on firms’ outsourcing decision, this study is inspired by two seminal papers (Feenstra and Hanson 1996; and Grossman and Rossi-Hansberg 2008). Both papers study a firm’s decision on how to source intermediates (or tasks): whether from home suppliers or abroad. The decision on how to source intermediates is based on the relative cost between the two; a firm sources each intermediate from the lowest-cost supplier. The present study extends the previous work mainly in two respects: first, it incorporates quality differences between suppliers into the firm’s decision of where to outsource; second, it allows a firm to vertically differentiate its product across markets by sourcing each intermediate from multiple suppliers of different quality. To my knowledge, such extensions, which appear to be useful in understanding Southern firm’s importing strategy, have not been explored so far in the literature.

Two recent empirical findings motivate this study: quality-to-market and increasing importance of intermediate trade. A number of studies provide econometric evidence that supports the validity of quality-to-market at the firm-level. They find that firms charge higher factory-gate (fob) prices for their products (i) in rich than in poor markets, and (ii) in distant than in near markets. Both findings are found to be stronger for differentiated products. These are robust findings as they have been confirmed by studies using data from countries with quite different characteristics: Bastos and Silva (2010) using data from Portugal; Manova and Zhang (2012) from China; and Görg, Halpern, and Muraközy (2010) from Hungary. Crozet, Head, and Mayer (2012) report similar findings for French wine exporters. Assuming that high quality is associated with high price, (i) suggests that a firm sells high-quality products to its customers located in rich markets. With respect to (ii), the finding suggests that a firm-level Alchian-Allen effect is at work: distance raises transport costs, and a higher (per unit) transport cost is associated with a lower relative price of high-quality to low-quality products, resulting in a higher relative demand for the high-quality
one. In other words, the firm “ships the good apples out”. These two observations (i) and (ii) point to the quality-to-market hypothesis as a plausible explanation.

Another stylised fact that motivates the current paper is the recent increase in intermediate trade. Imported inputs can provide Southern firms access to high-quality inputs. Kugler and Verhoogen (2009) use transaction-level data from a Southern country, Colombia, and present evidence pointing to the existence of quality differences between domestic and imported varieties of inputs. At the firm-product-level, they find that Colombian firms pay higher prices for imported inputs than for domestic inputs. After exploring other possible reasons for the observed differences between domestic and imported input varieties, Kugler and Verhoogen conclude that quality difference appears as the most plausible one. Manova and Zhang (2012) use transaction-level data from China and find that firm-product-level variation in input prices is positively associated with firm-product-level variation in export prices. They interpret the findings suggesting that Chinese firms that export multiple-quality varieties source their inputs from multiple countries. This implies that sourcing inputs from multiple countries is a means of product differentiation. The mechanism is not specific to China. Goldberg et al. (2009, 2010) provide evidence that Indian firms also benefited from imported inputs to increase their product diversity in the aftermath of trade liberalisation. In another paper, Feng, Li, and Swenson (2012) use Chinese data and find that an increase in the variety of imported intermediates, particularly from OECD countries, led to an increase in the variety of Chinese exports in the period 2002-2006.

The paper also studies a firm’s decision to import. The model predicts that only more productive firms select into importing. Gibson and Graciano (2011) derive a similar finding, relying on the presence of fixed cost of importing. Here the result arises because importing intermediates raises a firm’s marginal cost, and the cost is worthwhile only if using imported intermediates results in a sufficient increase in the firm’s marginal revenue. In the current model, only for sufficiently productive firms the resulting increase in marginal revenue is large enough to overcome the resulting increase in marginal cost due to imported in-

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2 Hummels and Skiba (2004) build a simple model to show that a country changes the relative share of its high-quality exports owing to per unit transport costs; they also provide empirical support for the Alchian-Allen hypothesis.
The paper is also related to the recently growing theoretical literature on quality and firm heterogeneity. This literature tells us that high-productivity firms produce high-quality products, examples include Baldwin and Harrigan (2011), Johnson (2011), Kneller and Yu (2008), Verhoogen (2008), and Kugler and Verhoogen (2011). Verhoogen (2008) and Kugler and Verhoogen (2011) put forward input quality as a source of quality differentiation between firms—it is labour and capital in Verhoogen (2008) and a single domestic intermediate input in Kugler and Verhoogen (2011). The current study contributes to the literature by proposing a new mechanism for quality differentiation between firms: more productive firms produce higher quality products by using more variety of imported intermediates, which are of higher quality compared to the domestic varieties.

The rest of the paper is organised as follows. Section 1 introduces the model. Section 2 considers a Southern firm’s decisions on what to import and export, and the determinants of these decisions. The following section studies the firm’s decision of whether to export and import. Section 4 concludes.

1 Model

Throughout the paper the home country is a Southern country. It is populated by people who inelastically supply their skills. Suppliers employ skills in the production of intermediates, and firms use intermediates in the production of final goods. Until Section 3, I assume multiple destination markets for the goods produced by home country firms.

1.1 Consumers

A two-tier utility function represents consumer preferences in destination market $c$. The upper-tier is a Cobb-Douglas function which determines the allocation of a consumer’s budget between a homogeneous good ($x_{c,0}$) and a continuum of horizontally (and vertically) differentiated varieties indexed by $\phi$. The lower-tier is a CES aggregate of differentiated goods, where quality ($q(\phi)$) augments quantity ($x(\phi)$). The specification of the utility function follows the one introduced by Hallak (2006):
\begin{equation}
U_c = x_{c0}^{1-\mu} \left[ \int_{\phi \in \Omega_c} \frac{q_c(\phi)^{\gamma_c} x_c(\phi)}{\sigma} d\phi \right]^{\frac{\mu \gamma_c}{\sigma}} ; \sigma > 1, 0 < \gamma_c \leq 1, 0 < \mu < 1;
\end{equation}

where $\mu$ denotes budget share of differentiated goods, $\sigma$ elasticity of substitution between varieties, and $\gamma_c$ intensity of consumer preferences for quality in destination market $c$. In (1), $\Omega_c$ gives the set of available varieties of the differentiated good in $c$.

The specification in (1) incorporates heterogeneous consumer preferences for quality since the parameter $\gamma$ is destination-specific. Hallak (2006) empirically finds that $\gamma$ is increasing in consumer income: richer consumers have more intense preferences for quality.\footnote{Hallak (2003) shows that a similar relationship between income and quality choice can be derived from a nested logit model of demand. Nevertheless, the specification in (1) proves to be more tractable.} Hallak’s finding is also consistent with the demand side of the Linder hypothesis (1961). Linder argues that countries with higher per capita income demand relatively higher-quality goods.

### 1.2 Suppliers

A supplier is an intermediate producer. As in Feenstra and Hanson (1996) and Grossman and Rossi-Hansberg (2008), production of a final good requires a continuum of intermediates (or tasks) that can be sourced either from the home country or abroad. Grossman and Rossi-Hansberg (2008) use the term "task", instead of "intermediate", to describe the 21st century trade in intermediate goods. Rapid improvements in information and communication technologies have eased the coordination of activities across different geographic areas, and have broken down production into smaller tasks. As a result, trading tasks that used to be non-tradable, such as accounting, can now be traded. I use the two terms interchangeably in the paper.

In both above-mentioned studies, the decision of where to source intermediates is based on the relative cost between the two. The current model incorporates quality differences between home and foreign-supplied intermediates. Therefore, in addition to relative cost differences, a final good producer should also take quality differences into account when deciding where to source each
intermediate. This feature distinguishes the current analysis from the other two studies. Furthermore, it provides a flexible supply mechanism that allows final good producers to differentiate the quality of their products across markets.

As mentioned above, production of a final good consists of a continuum of intermediates indexed by \( j, j \in [0, 1] \). Any intermediate can be sourced from domestic suppliers or imported from Northern countries. Suppliers operate in perfectly competitive industries. Domestic and Northern suppliers produce vertically (quality) differentiated varieties of each intermediate. To be specific, for each intermediate, domestic variety is of lower quality than the imported variety.\(^4\) Depending on its choice of product quality, a final good producer decides from which supplier to source each intermediate. This tractable supply mechanism allows us to study within-firm product differentiation by quality.

Intermediates lie on the unit interval such that their skill requirements are increasing; let \( a_j \) denote the skill requirement of intermediate \( j \), then \( a_j > 0 \).\(^5\) Northern skill is equally productive as domestic skill in the physical production of intermediates. But it is more productive than the domestic one in the quality production: one unit of Northern skill produces one unit of quality, and one unit of domestic skill produces \( \lambda \) units of quality, \( \lambda < 1 \).\(^6\) The parameter \( \lambda \) can be thought of as the productivity of domestic labour relative to Northern labour.

There is a large number of suppliers in each industry. So, a supplier charges a price that is equal to its marginal cost of production. It implies that a Northern supplier of intermediate \( j \) charges a price equal to

\[
p_j^N = a_j r_N,
\]

and a domestic supplier charges a price equal to

\[
p_j^S = a_j r_S,
\]

where \( r_N \) and \( r_S \) denote the price of skill in the North and in the home country, respectively. Since Northern skills are more productive, their unit price is higher.

\(^4\)There can be other Southern countries where intermediate suppliers are located. I implicitly assume that, for any intermediate, the lowest-cost supplier of the low-quality variety is located in the home country.

\(^5\)Indeed, skill requirement refers to efficiency labour requirement.

\(^6\)See Clark (1987) for a discussion about the possible reasons of differences in labour quality between the North and the South.
higher: \( r_N > r_S \). There are iceberg-type trade costs \( \tau > 1 \) on intermediates, thus the domestic price of intermediate \( j \) imported from the North is equal to \( \tau a_j r_N \).\(^7\)

### 1.3 Firms

A firm is a final good producer. There is a large number of firms that operate in a monopolistically competitive industry. A firm pays a fixed cost to run the production facilities and can use them to manufacture different (vertically differentiated) varieties of a product. In other words, the fixed cost of introducing a new (vertically differentiated) variety is negligible.\(^8\) To produce a variety of the final good, the firm combines an equal amount of each intermediate.

I use the following production function specification, as introduced by Kugler and Verhoogen (2011):

\[
F(n) = n\phi^\alpha, \tag{4}
\]

where \( n \) denotes the number of each intermediate, \( \phi > 1 \) firm productivity, and \( \alpha > 0 \) sensitivity of unit cost to firm productivity.\(^9\) Thus, a firm with productivity \( \phi \) requires \( \phi^{-\alpha} \) units of each intermediate to produce one unit of final good. Its marginal cost of production is lower the higher the sensitivity of costs to firm productivity (higher \( \alpha \)). Given (4), the marginal cost of a firm with productivity \( \phi \) is equal to

\[
C(\phi, P_{int}) = \phi^{-\alpha}P_{int}, \tag{5}
\]

where \( P_{int} \) is the total cost of intermediates per unit of production.

The quality of the firm’s product is a function of its productivity (\( \phi \)) and the quality of the intermediates it uses in production (\( \Psi \)). I assume that \( q(\phi, \Psi) \) has

\(^7\)One can also assume a mix of specific and ad valorem trade costs for inputs. Since I am interested particularly in trade costs for final goods, I keep trade costs for intermediates as simple as possible.

\(^8\)As in Bernard, Redding, and Schott (2010), varieties in a market are differentiated from each other by their brand. A firm is allowed to supply only one horizontal variety in a market, although it is allowed to produce different vertically-differentiated varieties to sell in different markets. So, throughout the paper, \( \phi \) is an index for both firms and brands.

\(^9\)The production function in (4) can be derived from the O-ring production function, developed by Kremer (1993), if the capital requirement is set to unity and all tasks are performed perfectly.
the following properties:

\[
(i) \frac{\partial q(\phi, \Psi)}{\partial \Psi} > 0; (ii) \frac{\partial^2 q(\phi, \Psi)}{\partial \Psi^2} < 0.
\]

Assumption (i) guarantees that product quality is increasing in the overall quality of the intermediates, the next one implies decreasing returns to intermediates quality for a given productivity. I do not impose any assumptions on the dependence of \(q(\phi, \Psi)\) on firm productivity or on the sign of the cross-partial derivatives. For instance, Kugler and Verhoogen (2011) impose log-supermodularity on \(q(\phi, \Psi)\), and Verhoogen (2008) assumes complementarity between productivity and input quality to obtain the result that more productive (capable) firms use higher quality inputs. In sub-section 2.3, I will show that even in the absence of such restrictions on \(q(\phi, \Psi)\), the result can follow. In other words, the result holds under more general conditions than previously considered.

The overall quality of the intermediates is measured by a weighted-average of their quality, where weights are the corresponding skill requirements. Therefore, if a firm sources intermediates in \([0, I]\) domestically and imports the rest from the North, the overall quality of its intermediates is equal to

\[
\Psi = \lambda \int_0^I a_j d j + \int_I^J a_j d j, \text{ or } \Psi = \lambda A + (1 - \lambda) A_N,
\]

where \(A = \int_0^1 a_j d j\) and \(A_N = \int_I^J a_j d j\). In words, the higher the share of intermediates sourced from Northern suppliers the higher is the overall intermediates quality:

\[
\frac{\partial \Psi(A_N)}{\partial A_N} = 1 - \lambda > 0.
\]

To sum up, firms combine a continuum of intermediates that they source from domestic or Northern suppliers, and produce differentiated goods in a monopolistically competitive industry. A firm has to incur an ad valorem trade cost \(\tau_{oc} > 1\) and a specific trade cost \(t_c > 0\) when it sells its product to market \(c\). So, if the price charged by the firm is \(p\), the price faced by a consumer located in market \(c\) is \(p_{ci}^c(\phi) = \tau_{oc} p(\phi) + t_c\)\(^{10}\).

\(^{10}\)Hummels and Skiba (2004) use the same specification for consumer prices.
2 Partial Equilibrium

In this section, I focus on the behaviour of a single firm with productivity $\phi$. I assume that the firm uses a mix of domestic and imported intermediates ($0 < A_N < A$) and exports its product to multiple destinations. I will study the firm’s decision to import and export in Section 3.

The firm sources intermediates in $[0, I]$ from Southern suppliers and the rest from Northern suppliers. Given (5), its marginal cost of production is equal to

$$C(\phi, A_N) = \phi^{-\alpha} [r_S A + (\tau r_N - r_S) A_N].$$  \hfill (8)

Demand it faces in region $c$ is

$$x_c(\phi) = \frac{\mu Y_c}{P_c} (q(\phi, \Psi)c)^{\gamma c - 1} \left( \frac{p_{c}^{\text{cif}}(\phi)}{P_c} \right)^{-\sigma},$$  \hfill (9)

where $P_c = \left[ \int_{\phi \in \Omega_c} \left( \frac{p_{c}^{\text{cif}}(\phi)}{q_c(\phi, \Psi)c} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$ denotes quality-adjusted price index, and $Y_c$ aggregate income in region $c$.

Given the demand for its product, the firm chooses the price of its product ($p$) and the fraction of intermediates to be imported ($A_N \in (0, A)$). Its choice of $A_N$ determines the marginal cost of production by (8), and the quality of the final good by (7). Here is the profit maximisation problem of the firm: \footnote{Here, I drop region subscripts.}

$$\max_{p(\phi), A_N \in (0, A)} \Pi(\phi, A_N) = x(\phi, A_N) (p - C(\phi, A_N)).$$

Appendix A.1 formulates the Lagrangian corresponding to this problem, derives the first-order conditions, and checks the second-order conditions for a maximum. I consider the possibility of boundary solutions, i.e. decision to import, in Section 3. In the current section, I derive comparative statics at an interior solution. The resulting first-order conditions with respect to $p$ and $A_N$ are

$$Bq^{\gamma(\sigma-1)}(p_{c}^{\text{cif}})^{-(\sigma+1)} [-\sigma \tau_o (p - C) + \tau_o p + t] = 0$$  \hfill (10)

$$Bq^{\gamma(\sigma-1)-1}(p_{c}^{\text{cif}})^{-\sigma} [\gamma(\sigma - 1) (p - C) \frac{\partial q}{\partial A_N} - q \frac{\partial C}{\partial A_N}] = 0.$$  \hfill (11)
where $B = \mu Y P^{\sigma - 1}$.

Using the first-order condition with respect to price, we can see that the firm charges the following price to maximise its profits:

$$p(\phi) = \frac{1}{\sigma - 1} \left( \sigma C(\phi, A_N) + \frac{t}{\tau_o} \right), \text{ so } \quad (12)$$

$$p^cif(\phi) = \frac{\sigma}{\sigma - 1} (\tau_o C(\phi, A_N) + t). \quad (13)$$

The price set by a firm equals to a constant mark-up over its marginal cost, plus a fraction of the ratio of specific to ad valorem trade costs. So, the firm charges a higher price in a market where it sells a variety produced at a higher cost, and where it incurs higher specific to ad valorem trade costs. If specific trade costs are zero ($t = 0$), then producer prices are independent of ad valorem costs. On the other hand, even in the absence of ad valorem trade costs ($\tau_o = 1$), producer prices depend on specific trade costs.

To maximise its profits, the firm also chooses the fraction of its imported intermediates $A_N^*$. The condition in (11) implies, if $A_N^* \in (0, 1)$, then it solves the following equation:

$$\gamma (\sigma - 1) (p - C) \frac{\partial q(\phi, A_N)}{\partial A_N} - q \frac{\partial C(\phi, A_N)}{\partial A_N} = 0. \quad (14)$$

In the absence of quality considerations, minimising costs would be the only motive determining the firm-level intermediate trade. It is what we learn from the model developed by Feenstra and Hanson (1996). In the current model, on the other hand, when deciding on the extent of its intermediate trade, a firm faces a trade-off between two opposing effects. A higher $A_N$ increases the firm’s marginal cost:

$$\frac{\partial C(\phi, A_N)}{\partial A_N} = \phi^{-\alpha (\tau r_N - r_S)} > 0.$$ 

At the same time, a higher $A_N$ improves the quality of its product:

$$\frac{\partial q(\phi, A_N)}{\partial A_N} = \frac{\partial q(\phi, I)}{\partial \Psi} (1 - \lambda) > 0,$$
and thus increases the demand for its product:

$$\frac{\partial x(\phi, A_N)}{\partial A_N} = \frac{\partial x(\phi, I)}{\partial q} \frac{\partial q(\phi, I)}{\partial A_N} > 0.$$  

In the rest of this section, I characterise the comparative statics of $A_N$—the firm’s profit maximising level of intermediate trade—with respect to the intensity of preferences for quality ($\gamma$), trade costs for both intermediates ($\tau$) and final goods ($\tau_o, \tau$), and firm productivity ($\phi$). Let us define the set $\mathcal{F} = \{\gamma, \tau, \tau_o, \tau, \phi\}$. The change in $A_N$ as a response to any exogenous parameter $\delta \in \mathcal{F}$, is given by

$$\frac{\partial A_N}{\partial \delta} = \frac{\Pi_{A_N} \delta \Pi_{pp} + \Pi_p \delta \Pi_{Ap}}{\Pi_{pp} \Pi_{A_N} - \Pi_p \Pi_{Ap}}, \quad (15)$$

where $\Pi_{A_N} \delta = \frac{\partial^2 \Pi}{\partial \delta \partial A_N}$, and other cross-partial are defined accordingly. Appendix A.1 derives the second-order condition for a maximum, and the condition implies that the denominator in (15) is positive: $\Pi_{pp} \Pi_{A_N} - \Pi_p \Pi_{Ap} > 0$. So, we have

$$\frac{\partial A_N}{\partial \delta} \propto -\Pi_{A_N} \delta \Pi_{pp} + \Pi_p \delta \Pi_{Ap}. \quad (16)$$

Appendix A.1 shows that $\Pi_{pp} < 0$ and $\Pi_{Ap} > 0$. I will use (16) to derive the comparative statics for the firm-level intermediate trade.

### 2.1 Effect of consumer preferences

What is the effect of the intensity of consumer preferences for quality $\gamma$ on the firm’s outsourcing decision—the fraction of its imported intermediates ($A_N^*$)? To answer this question, we can use (16) where $\delta = \gamma$:

$$\frac{\partial A_N^*}{\partial \gamma} \propto -\Pi_{A_N} \gamma \Pi_{pp} + \Pi_p \gamma \Pi_{Ap}. \quad (17)$$

Given (10) and (11), we have

$$\Pi_{A_N} = Bq^{\gamma(\sigma-1)-1} \left(p^{ci}f\right)^{-\sigma} (\sigma - 1) (p - C) \frac{\partial q}{\partial A_N} > 0,$$

$$\Pi_p = 0.$$

Then, since $\Pi_{pp} < 0$, we obtain
\[ \frac{\partial A_N^*}{\partial \gamma} = -Bq^{\gamma(\sigma-1)-1}\left(p^{\epsilon_f}\right)^{-\sigma}(\sigma - 1)(p-C) \frac{\partial q}{\partial A_N} \Pi_{pp} > 0. \]  

Consider two markets \( c \) and \( c' \) such that \( \gamma_c > \gamma_{c'} \); the firm faces more intense preferences for quality in \( c \) than in \( c' \). The inequality in (17) implies that the firm sources a higher fraction of intermediates from Northern suppliers to produce and sell a higher-quality variety in \( c \):

\[
\left( \frac{dq(\phi, A_N)}{d\gamma} \right)_{A_N = A_N^*} = \left\{ \frac{\partial q(\phi, A_N)}{\partial A_N} \frac{dA_N}{d\gamma} \right\}_{A_N = A_N^*} > 0,
\]

and it charges a higher price in that market:

\[
\left( \frac{dp(\phi)}{d\gamma} \right)_{A_N = A_N^*} = \frac{\sigma}{\sigma - 1} \left\{ \frac{\partial C(\phi, A_N)}{\partial A_N} \frac{dA_N}{d\gamma} \right\}_{A_N = A_N^*} > 0.
\]

Now, let us write \( \frac{\partial A_N^*}{\partial \gamma} \) explicitly using the expressions for \( \Pi_{pp} \) and \( |H| \) derived in Appendix A.1, and simplify the resulting expression to obtain:

\[
\frac{\partial A_N^*}{\partial \gamma} = \frac{\sigma}{q(dC/dA_N) \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2} + \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N}(1-\gamma)}.
\]

Thus, we have \( \frac{\partial^2 A_N^*}{\partial (\gamma/\tau_o) \partial \gamma} > 0 \). A Southern firm uses more variety of high-quality imported intermediates when supplying a market where consumers have greater demand for quality. The incentive to produce a higher quality variety for a high-demand market is greater the higher the importance of specific relative to ad valorem trade costs associated with exporting. Since a high \( t/\tau_o \) leads to higher mark-ups, we can restate the previous result as follows: the incentive to produce a higher quality variety for a high-demand market is greater for products with higher mark-ups.

The following result summarises these findings.

**Result 1** A Southern firm facing different preferences for quality in different markets differentiates the quality of its product; it sells higher-quality varieties in high-demand markets. The firm produces high-quality varieties by using more
variety of high-quality imported intermediates from the North. The incentive to upgrade quality as a response to high demand for quality increases with the ratio of specific to ad valorem costs.

The firm, facing different demand for quality in different markets, chooses endogenously to become a multi-product firm. This firm produces multiple varieties (versions) of a product in a single product line, and sells them in different markets. The firm engages in quality differentiation by changing the fraction of high-quality imported intermediates across its varieties. Since it pays a higher price for an imported intermediate, the firm’s production cost varies between the varieties. As a result, the price it charges also changes across markets.

Result 1 is in line with the empirical findings of Bastos and Silva (2010), Manova and Zhang (2012), and Görg, Halpern, and Murakózy (2010): a Southern firm charges higher factory-gate prices for its product in rich than in poor markets. Among these studies, Manova and Zhang (2012) is the only study that provides a mechanism for the observed pattern of firm-product level prices across destinations. Their findings confirm the mechanism suggested by Result 1: Chinese firms that differentiate the quality of their products use intermediates of different quality levels.

The result highlights a link between firm’s extensive intermediate imports margin and its product margin. As Southern firms gain access to new markets where quality demand is higher than in their domestic market, they add new product varieties to their product lines. Since they sell the new varieties in the rich markets, they should be of higher quality than the existing ones. To produce them, firms need high-quality input varieties that may not be available in the domestic market. They can source such inputs from the North as Northern workers are able to produce higher-quality inputs than their Southern counterparts. In other words, Southern firms gain access to high-productivity Northern workers through intermediate imports. Thus, an expansion on Southern firms’ extensive product margin towards higher-quality varieties is associated with an expansion on their extensive intermediate imports margin towards higher-quality inputs.

I use the term here in a different sense from the existing trade literature that studies firms that produce horizontally-differentiated products (e.g. Bernard, Redding, and Schott 2010; Eckel and Neary 2010), and from the industrial organisation literature that studies firms that produce vertically differentiated products for a single market (e.g. Johnson and Myatt 2003, and the studies cited in that paper). Here, I study firms that produce vertically differentiated products for segmented markets.
This mechanism finds empirical support. Goldberg et al. (2009, 2010) provide evidence that Indian firms benefited from imported inputs to increase their product diversity in the aftermath of trade liberalisation. Furthermore, they report that almost 70 percent of the growth in the extensive margin of intermediate imports by Indian firms originated from OECD countries. These intermediates can be assumed to be of higher quality compared to the domestic varieties.

2.2 Effect of trade costs

In the model, there are three types of trade costs: an iceberg-type cost on intermediates $\tau$, an ad valorem cost on final goods $\tau$, and a specific (per unit) cost on final goods $t$. A firm uses a mix of domestic and imported intermediates, and the imports content of the mix depends on the relative price of imported intermediates and the firm’s choice of product quality. On the one hand, anything that affects the relative price of imported intermediates will affect the firm’s product quality and the volume of exports. On the other hand, anything that affects the firm’s incentive to upgrade the quality of its product will affect the volume of intermediate imports.

Let us start with discussing the effect of a fall in the cost of importing intermediates on product quality. Intuitively, this will reduce the cost of imported intermediates relative to the domestic varieties, inducing firms to increase their use of imported intermediates. Therefore, the quality of all varieties produced by a firm will increase. To derive the sign of $\frac{\partial A_N^*}{\partial \tau}$ we need to use the following:

\[ \Pi_{A_N^*} = -Bq^{\gamma}(\sigma-1)^{-1} \left( p_c^{\text{iff}} \right)^{-\sigma} \left[ \gamma(\sigma-1) \frac{\partial C}{\partial \tau} \frac{\partial q}{\partial A_N} + q \frac{\partial^2 C}{\partial \tau \partial A_N} \right] < 0, \]

\[ \Pi_{p^*} = Bq^{\gamma}(\sigma-1) \left( p_c^{\text{iff}} \right)^{-\sigma-1} \frac{\partial C}{\sigma \tau_0 \tau^2} > 0, \]

where $\frac{\partial C}{\partial \tau} = \phi^{-\alpha} r_N A_N$ and $\frac{\partial^2 C}{\partial \tau \partial A_N} = \phi^{-\alpha} r_N$. Using the expression in (15), Appendix A.2 shows that

\[ \frac{\partial A_N^*}{\partial \tau} < 0. \]
Furthermore, we can easily see that

\[
\left( \frac{dq(\phi, AN)}{d\tau} \right)_{AN=A_N^*} = \left\{ \frac{\partial q(\phi, AN)}{\partial AN} \frac{dAN}{d\tau} \right\}_{AN=A_N^*} < 0.
\]

This result is not surprising, and in line with the intuition described above. A Southern firm upgrades the quality of all varieties it produces when the cost of importing high-quality intermediates from the North falls. An interesting result concerns the dependence of this quality-upgrading effect on the characteristics of the destination market:

\[
\frac{\partial AN^*_N}{\partial \tau} = \sigma r_N \phi^\alpha r_S A + s_{\tau r_N}^T + \left\{ \frac{\partial C(\phi, \Psi)}{\partial AN} \frac{dAN}{d\tau} \right\}_{AN=A_N^*} \gamma \left( 1 + \gamma (\sigma - 1) \right) + \sigma < 0.
\]

A reduction in the cost of importing creates an incentive for the firm to upgrade the quality of its product by importing more variety of high-quality intermediates; the incentive increases with the intensity of consumer preference for quality in the destination market, and with the share of specific to ad valorem costs incurred when exporting the product. So, those firms that already export to such destinations will benefit more from trade liberalisation in intermediates. This prediction is novel, and needs to be confirmed by data.

We know from Result 1 that the marginal cost of producing a variety exported to a high-demand market is high. According to expression (12), a firm’s absolute mark-up is higher the higher the marginal cost or the higher the ratio of specific to ad valorem trade costs. Then the discussion above suggests that a fall in the cost of importing high-quality intermediates encourages quality upgrading of products with higher absolute mark-ups more significantly. The following result summarises these findings.

**Result 2** A fall in the cost of importing intermediates induces firms to increase their use of high-quality imported intermediates, leading to quality upgrading. The incentive to upgrade quality as a response to a fall in \(\tau\) increases with the demand for quality and the ratio of specific to ad valorem costs.

There is empirical evidence that a firm’s factory-gate (fob) prices increase with distance (Görg, Halpern, and Muraközy 2010; Bastos and Silva 2010;
Manova and Zhang 2012; and Martin 2012). This empirical finding is interpreted to be closely related to Alchian-Allen’s “shipping the good apples out” hypothesis [1964], which argues that adding a per unit transport cost lowers the relative price of high-quality products, and thus increases their demand relative to low-quality products. It is a demand-side explanation. We need to complement it with a supply mechanism. It is where the current model comes in.

It is usually assumed that per unit trade cost is increasing in bilateral distance. Assuming so, a model with linear demand implies a negative correlation between firm-level fob prices and distance; a firm reduces its mark-up in a more distant market. In a model with logit demand, as in Verhoogen (2008), a firm lowers both the price and the quality of its product in more distant markets (Martin 2009). So, models with linear and logit demand predict the opposite of what is observed in the data. In a model with CES preferences, a firm adds to its fob price a fraction of the cost it bears to transport the product (see (12)). So, it charges a higher price in a more distant market.

Although a model with CES preferences and specific trade costs predicts what is observed in the data, the explanation is not complete. In addition to a positive correlation between fob prices and distance, two recent empirical studies (Görg, Halpern, and Muraközy 2010; Manova and Zhang 2012) also report that the observed correlation is stronger for differentiated products. This brings to mind the possibility that, in line with Alchian-Allen’s hypothesis, firms might be selling higher-quality products to bilaterally more distant markets. If high quality commands a high price, we would expect the correlation between fob prices and distance to be stronger when there is room for quality differentiation.

To see what the model says about the effect of distance on the firm’s product quality and price, let us use (15), noting that:

\[
\Pi_{pt} = B q^{\gamma (\sigma - 1)} \left( p^{clf} \right)^{-\sigma - 1} > 0
\]

\[
\Pi_{AN} = 0
\]

So,

\[
\frac{\partial A_N}{\partial t} = -\frac{\sigma}{\tau_0} \left\{ \frac{q}{\gamma} \frac{\partial C}{\partial A_N} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2} + \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] \right\} > 0.
\]

A Southern firm uses a higher fraction of imported intermediates from the North, and thus produces a higher-quality variety to sell in a distant than in a near
market. The firm charges a higher fob price in such market for two reasons: first, it adds part of the transport cost to its mark-up, and second, it uses more expensive intermediates to produce a higher-quality variety. A higher specific trade cost raises the relative demand for high-quality to low-quality varieties on the demand-side, and the firm’s mark-up on the supply-side. Thus, the model explains the observed correlation between fob prices and distance from both demand and supply sides.

Specific costs are not the only trade costs that an exporter has to pay in the model; there is also an ad valorem cost denoted by $\tau_o$. A firm’s mark-up increases with $\tau$ but decreases with $\tau_0$. Therefore, it is reasonable to expect ad valorem costs to have a different effect on product quality and the extent of firm-level intermediate imports. To show this, use (15) and the following

$$\Pi_{p \tau_0} = -B_q \gamma (\sigma - 1) \left( p_{cif} \right)^{-\sigma - 1} \frac{t}{\tau_0} < 0$$

$$\Pi_{A_N \tau_o} = 0,$$

to obtain

$$\frac{\partial A^*_N}{\partial \tau_o} = \frac{t \sigma}{\tau_o} \left\{ q \frac{dC}{dA_N} \frac{d^2q(\phi, \psi)}{dA_N^2} + \frac{dC}{dA_N} \frac{dq}{dA_N} \left[ -\frac{1}{\gamma} (1 + \gamma(\sigma - 1)) + \sigma \right] \right\} < 0.$$

While an increase in specific costs increases firm-level intermediate imports for quality upgrading, an increase in ad valorem costs reduces it. A Southern firm uses less variety of imported intermediates to produce a variety for a market with high ad valorem costs. So, a Southern exporter charges a lower fob price in the market with high ad valorem costs for two reasons: first, it reduces its mark-up, and second, it uses cheaper intermediates to produce a lower-quality variety. If both specific and ad valorem costs increase with distance, then the correlation

13 There is an analogy between this result and the response of a firm to voluntary export restraints (VERs). An exporting country sets VERs to limit the quantity of its exports to a specific country. We know from other studies that a VER may also create an incentive for exporters to upgrade the quality of their products. In 1981, the Japanese government imposed a restriction on the number of its car exports to the U.S. This led to an increase in the price of Japanese cars imported by the U.S. Feenstra (1984) empirically illustrates that the two-thirds of this price increase resulted from quality improvement by the Japanese car exporters. Although the underlying mechanisms are different in two cases, this is an interesting analogy to note. I would like to thank Richard Baldwin for bringing this analogy to my attention.
between firm-level fob prices and distance will depend on their relative importance. Also the relative importance of specific to ad valorem trade costs for final goods has different implications for firm-level imported intermediates.

**Result 3** An increase in specific trade costs on final goods increases firm-level imported intermediates from the North, while an increase in ad valorem costs reduces them.

### 2.3 Firm productivity and intermediate imports

A Southern firm trades off the cost of imported intermediates against their contribution to product quality. An interesting question is how this trade-off changes with firm productivity. The properties of \( q(\cdot, \cdot) \), as stated in (6), do not tell us anything about the dependence of product quality on firm productivity. Comparative statics will tell us how a firm’s product quality depends on its productivity.

There is empirical evidence that more productive firms use higher-quality inputs. For instance, using firm-level Colombian data, Kugler and Verhoogen (2012) find that more productive Colombian firms pay higher prices for their material inputs, and the correlation holds even within narrow product categories. After running a number of robustness checks, the authors interpret the observed correlation as suggesting that more productive firms use higher-quality inputs. They rationalise this finding by extending Melitz’s heterogeneous firms model in two alternative ways: first by assuming that producing a high-quality final good requires a fixed cost and high-quality inputs, and second by imposing complementarity between input quality and firm productivity. In the latter variant, Kugler and Verhoogen assume that \( q(\cdot, \cdot) \) is log-supermodular in firm productivity and input quality. In other words, the elasticity of product quality with respect to input quality is increasing in firm productivity.

Here, I do not assume any fixed costs for upgrading product quality or impose log-supermodularity on the function \( q(\cdot, \cdot) \). For a given firm productivity, the function is assumed to be increasing and concave in input quality, and input quality is increasing in the use of imported intermediates. So, one needs to find how \( A_N^+ \) changes with firm productivity to understand the relationship between
input quality and firm productivity. To do so, we need to derive the following;

\[ \Pi_{p\phi} = B\sigma q^{\gamma(\sigma - 1)} \left( p^{ci} \right)^{-\sigma - 1} \frac{\partial C}{\partial \phi} < 0 \]

\[ \Pi_{AN\phi} = Bq^{\gamma(\sigma - 1) - 1} \left( p^{ci} \right)^{-\sigma} \left\{ \gamma(\sigma - 1) - q \frac{\partial^2 q}{\partial A_N \partial \phi} (p - C) - \frac{\partial q}{\partial A_N} \frac{\partial C}{\partial \phi} - \frac{\partial q}{\partial \phi} \frac{\partial C}{\partial A_N} - q \frac{\partial^2 C}{\partial A_N \partial \phi} \right\} . \]

Using (10), (11), and given (15), \( \partial A_N^- / \phi \) is proportional to:

\[ \left( C + \frac{t}{\tau_o} \right) \frac{\partial C}{\partial A_N} \left( \frac{\partial^2 q}{\partial A_N \partial \phi} \frac{q}{\partial q / \partial A_N} - \frac{\partial q}{\partial \phi} \right) + q\alpha \phi^{-\alpha - 1} \frac{t}{\tau_o} (\tau_r - r_S). \] (18)

If the elasticity of product quality with respect to input quality, \( \varepsilon^q_{\Psi} \), is increasing in firm productivity, then the sign of the expression in (18) is positive. In other words, if \( q(\ldots) \) is log-supermodular then more productive firms import a wider range of their intermediates from the North; they use higher-quality intermediates. This result is similar to the one obtained by Kugler and Verhoogen (2012).

According to (18), the result that more productive firms import a wider range of their intermediates from the North follows even when \( \varepsilon^q_{\Psi} \) is not increasing in firm productivity, provided specific costs are non-zero. Therefore, log-supermodularity of \( q(\ldots) \) is sufficient but not necessary for the result.

**Result 4** If specific trade costs for final goods are non-zero and \( \frac{\partial \varepsilon^q_{\Psi}}{\partial \phi} \) is not too negative, more productive Southern firms use more variety of imported intermediates from the North. By doing so, compared to less productive firms, they produce higher-quality varieties of the final good. If specific trade costs are zero, then the result will follow provided \( \frac{\partial \varepsilon^q_{\Psi}}{\partial \phi} \) is positive.

The key to Result 4 is the presence of specific trade costs. On the demand-side, the presence of such costs reduces the relative price of high-quality products, and thus increases their relative demand. On the supply-side, it reduces the cost advantage of high-productivity firms, and thus encourages them to upgrade their product quality to preserve their market share. The result hinges on the cross partial derivative of the cost function with respect to \( A_N \) and \( \phi \) being negative; it is less costly for more productive firms to upgrade their product quality by importing more variety of intermediates from the North. As a result,
more productive firms use higher-quality intermediates. Result 4 generalises the mechanism suggested by Kugler and Verhoogen (2012).

3 Industry Equilibrium

Having studied a single Southern firm’s decision what to import and export, it is time to study its decision of whether to import and export.

Since I am not interested in firm entry and exit, I extend the model to heterogeneous firms based on Chaney (2008) rather than Melitz (2003). There are two regions: North and South. Let $L_c$ denote the population of region $c$, and the relative size of the two regions satisfies $L_S > L_N$. In both regions, an individual inelastically supplies one unit of skill. The homogenous good $x_0$ is produced under constant returns to scale using one unit of skill and serves as a numéraire. The good is freely traded between the two regions. In each region the industry is large enough so that each produces a strictly positive output of $x_0$. In particular, if region $c$ produces $r_c$ amount of the numéraire, the unit skill price in $c$ is $r_c$. So, $r_c$ also gives the skill productivity in the region. As I have done so far, I name the high-productivity region the North: $\frac{r_S}{r_N} = \lambda < 1$. A firm has to pay a fixed production cost $f > 0$ in terms of the numéraire to run the production facilities. There also exist fixed costs associated with exporting ($f_X$) and importing ($f_M$), and they are paid in terms of the numéraire.

In the differentiated goods industry, there is a continuum of firms, indexed by their productivity $\phi$. Firm productivities are drawn from a Pareto distribution $(G(\phi))$ with shape parameter $\nu$:

$$G(\phi) = 1 - \phi^{-\nu}, \text{ where } \phi \in [1, \infty) \text{ and } \nu > 2.$$ 

The density function corresponding to $G(\phi)$ is $g(\phi)$. As in Chaney (2008), the mass of potential entrants in region $c$ is proportional to the region size $r_cL_c$, and total net profits generated in the differentiated goods industry are re-distributed to individuals such that each receives $r_c$ shares of the total profits.

In line with the empirical findings of Hallak (2006), and given $r_N > r_S$, I

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14 In an earlier version of the paper, I extend the model to heterogeneous firms assuming fixed entry costs as in Melitz (2003). The main results presented here remain unchanged. The above-mentioned version can be accessed at http://www.economics.ox.ac.uk/research/WP/pdf/paper582.pdf.
assume $\gamma_N > \gamma_S$: Northern consumers have more intense preferences for quality than Southern ones. As in Section 2, product quality is a function of firm productivity and the overall quality of the intermediates the firm uses in production. I assume a Cobb-Douglas function for product quality as follows:

$$q(\phi, \Psi) = \phi^{z-1} z \Psi^1 \in (0, 1).$$  \hspace{1cm} (19)

The functional form in (19) implies that the elasticity of product quality with respect to input quality is constant and equal to $1 - z$. In particular, it does not depend on firm productivity. Thus I do not impose log-supermodularity on $q(\ldots)$.

I focus on Southern firms only. In the South, importing intermediates for production of exports are exempt from duties. However, importing intermediates for production of domestically sold goods is subject to duties, which are represented by $\tau$ as before. For simplicity, I assume that $\tau$ is prohibitively high so that Southern firms use only domestically-sourced intermediates to produce for the domestic market.

3.1 Quasi-autarky

I define quasi-autarky a state where the two regions trade the homogenous good, but there is no trade in intermediates or in the differentiated good. The conditional factor price equalisation still holds: $r_S/r_N = \lambda$.

Since there is no trade in intermediates, $A_N = 0$ for any Southern firm. Given (5) and (7), and normalising $A = \int_0^1 a_j d j = 1$, the marginal cost of production and product quality of a Southern firm with productivity $\phi$ are given by

$$C(\phi) = \phi^{-\alpha} r_S,$$

$$q(\phi) = \phi^z \lambda^{1-\z}.$$

The domain of $G(\phi)$ is not the same as the set of active firms; only those firms that are able to earn enough (variable) profits to cover the fixed cost $f$ are active in the differentiated goods industry. The least productive active firm in the South, $\phi^*$, is determined by the following condition:

$$\Pi^{var}(\phi^*) - f = 0.$$
Appendix A.3 shows that the productivity of the least productive firm in the South is given by

$$\phi^* = \frac{\sigma}{\mu} \frac{v}{v - \theta_S} \frac{f}{1 + \pi},$$  \hspace{1cm} (20)

where $\theta_c = (\sigma - 1)(\alpha + \gamma_c z)$, and $\pi$ denotes dividend per share. The expression tells us that, all else equal, an increase in consumer demand for quality raises the cut-off productivity for producing for the domestic market. In particular, the cut-off productivity for serving the Northern market, $\phi^*_N$, is higher than that for serving the Southern market:

$$\frac{\phi^*_N}{\phi^*} = \frac{v - \theta_S}{v - \theta_N} > 1.$$ 

The inequality follows because $\theta_N > \theta_S$ given $\gamma_N > \gamma_S$.

One implication of the inequality is that an average Northern firm is more productive than an average Southern firm. Also, quality of an average final good variety sold in the Northern market is higher for two reasons: first, inputs available in the North are of higher quality, and second, Northern firms are, on average, more productive.

**Result 5** An average Northern firm is more productive than an average Southern firm. It follows that an average final good variety on the Northern market is of higher quality.

Result 5 is closely related to the supply-side of Linder’s hypothesis (1961). Linder argues that countries have a comparative advantage in the products for which there is strong domestic demand. In the current model, Northern consumers demand higher-quality varieties than Southern consumers. The demand bias leads to a more stringent firm selection in the North. Therefore, active firms in the Northern market produce, on average, higher-quality varieties than Southern firms because of two reasons: first, they are more productive, and second they have access to higher-quality intermediates in the domestic market. So the model neatly demonstrates the supply-side of Linder’s hypothesis.

### 3.2 Decision to import and export

A Southern firm may also decide to sell its product in the North, and in doing so the firm may decide to use duty-free imported intermediates. Without imposing
a functional form on \(q(\ldots)\), Appendix A.1, forms the Lagrangian corresponding to the profit maximisation problem faced by a Southern firm. Assuming that product quality is represented by (19), the resulting first-order condition with respect to \(A_N\) and Complementary Slackness conditions are as follows

\[
B(1 - \lambda) q^{\sigma(p^{eff})^{-1}} \left( p^{eff} \right)^{-\sigma} \phi^z \Psi^{-z} \left[ \gamma_N (1 - z) \left( C + \frac{t}{\tau_o} \right) - \Psi \phi^{-\alpha} r_N \right] + y_1 - y_2 = 0, \tag{21}
\]

\[
y_1 A_N = 0; \quad y_1 \geq 0, \tag{22}
\]

\[
y_2 (A - A_N) = 0; \quad y_2 \geq 0; \tag{23}
\]

where \(y_1\) and \(y_2\) are the respective Lagrange multipliers.

When none of the constraints bind, we must have \(y_1 = y_2 = 0\). So, \(A_N^* \in (0, 1)\) must satisfy

\[
\gamma_N (1 - z) \left( \phi^{-\alpha} r_N \Psi(A_N^*) + \frac{t}{\tau_o} \right) - \Psi(A_N^*) \phi^{-\alpha} r_N = 0
\]

\[
\implies \Psi(A_N^*) = \frac{\gamma_N \frac{t}{\tau_o} (1 - z) \phi^\alpha}{r_N (1 - \gamma_N (1 - z))}. \tag{24}
\]

According to (24), comparative statics for \(A_N^*\), as derived in Section 2, are preserved; \(A_N^*\) is increasing in consumer demand for quality \(\gamma\) and specific trade costs \(t\), but decreasing in \(\tau_o\). Also, \(A_N^*\) increases when the elasticity of product quality with respect to input quality, \(1 - z\), increases. In other words, Southern firms increase their use of high-quality intermediates as product quality becomes more dependent on input quality. The parameter \(z\) can be industry-specific. So, in industries where quality of intermediates is an important determinant of final good quality, firms rely more on high-quality imported intermediates.

Given (7), the condition \(0 < A_N^* < 1\) is equivalent to \(\lambda < \Psi(A_N^*) < 1\). When the first constraint binds, \(A_N^* = 0\), we have \(y_1 > 0\), then the following must hold

\[
\phi^\alpha < \frac{\lambda r_N (1 - \gamma_N (1 - z))}{\frac{t}{\tau_o} \gamma_N (1 - z)}.
\]

Therefore, a Southern firm that uses imported intermediates in the production
of its exports must be sufficiently productive such that

$$\phi > \phi^*_M = \left( \frac{\lambda r_N (1 - \gamma_N (1 - z))}{\frac{1}{\tau_o} \gamma_N (1 - z)} \right)^{1/\alpha}.$$  

If a firm’s productivity satisfies

$$\phi > \left( \frac{r_N (1 - \gamma_N (1 - z))}{\gamma_N (1 - z) \frac{1}{\tau_o}} \right)^{1/\alpha},$$

then the firm uses only high-quality imported intermediates in the production of its exports. In the rest of the paper, I will not analyse this case further.\(^{15}\)

**Result 6** Regardless of whether there is a fixed importing cost, there exists a cut-off productivity for Southern firms to import intermediates from the North, which is equal to

$$\left( \frac{\lambda r_N (1 - \gamma_N (1 - z))}{\frac{1}{\tau_o} \gamma_N (1 - z)} \right)^{1/\alpha}.$$  

Result 6 implies that only more productive Southern firms use high-quality imported intermediates from the North in the production of their exportables. In other words, only more productive firms select into importing. The result is in line with the one derived by Gibson and Graciano (2011). Their result relies on the presence of fixed cost of importing. In the current model, the result arises because imported intermediates are expensive. The additional cost is worthwhile only if using imported intermediates results in a sufficient increase in the firm’s marginal revenue. In the model, all else equal, more productive firms produce higher quality varieties of the final good, and charge lower quality-adjusted prices. Thus, only for sufficiently productive firms the resulting increase in marginal revenue is high enough to overcome the resulting increase in marginal cost due to imported intermediates. Result 6 also implies that the cut-off productivity for importing intermediates from the North is lower for Southern firms that export to markets with high demand for quality or with high specific to ad valorem trade costs, or for those operating in industries where quality of intermediates, rather than firm productivity, is an important determinant of final good quality.

Consider a Southern firm’s decision to export to the North. The firm has to decide whether to use domestic intermediates only, or a mix of domestic

\(^{15}\)I will also assume that the productivity threshold for importing is binding.
and imported intermediates. If the firm uses domestic intermediates only, the resulting profits from its export sales are

$$\Pi_X(\phi) = \frac{B_N}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \lambda^{(\sigma-1)(1-z)} \left( \lambda r_N + \frac{t\phi\alpha}{\tau_o} \right)^{1-\sigma} \phi^{(\sigma-1)(\alpha+z)} - f_X.$$  

As a benchmark, assume that fixed cost of importing is zero, i.e. \(f_M = 0\). Then, if the firm uses a mix of domestic and imported intermediates, its resulting profits are

$$\Pi_{MX}(\phi) = \frac{B_N}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left( \frac{t\phi\alpha}{z\tau_o} \right)^{(1-\sigma)z} \left( \frac{r_N}{1-z} \right)^{(1-\sigma)(1-z)} \phi^{(\sigma-1)(\alpha+z)} - f_X.$$  

We can re-arrange these expressions as follows

$$\Pi_X(\phi) = \frac{B_N}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left( z + \frac{z t\phi\alpha}{\tau_o \lambda r_N} \right)^{1-\sigma} \phi^{(\sigma-1)(\alpha+z)} - f_X,$$

$$\Pi_{MX}(\phi) = \frac{B_N}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left[ \left( \frac{z}{1-z} \right)^{(1-\sigma)} \left( \frac{t\phi\alpha}{\tau_o \lambda r_N} \right)^z \right]^{1-\sigma} \phi^{(\sigma-1)(\alpha+z)} - f_X.$$  

It is easy to see the relationship between \(\Pi_X(\phi)\) and \(\Pi_{MX}(\phi)\): \(\Pi_X(\phi)\) is in terms of a weighted-arithmetic mean of \(\frac{z}{1-z}\) and \(\frac{t\phi\alpha}{\tau_o \lambda r_N}\), while \(\Pi_{MX}(\phi)\) is in terms of their geometric mean, where weights are given by \(1-z\) and \(z\):

$$z + \frac{z t\phi\alpha}{\tau_o \lambda r_N} = \left( 1 - z \right) \frac{z}{1-z} + z \frac{t\phi\alpha}{\tau_o \lambda r_N}.$$  

Since the geometric mean is smaller than the arithmetic mean, we have $^{16}$

$$z + \frac{z t\phi\alpha}{\tau_o \lambda r_N} > \left( \frac{t\phi\alpha}{\tau_o \lambda r_N} \right)^z \left( \frac{z}{1-z} \right)^{(1-z)}.$$  

$^{16}$The proof uses the weighted Jensen’s inequality: if \(f(x)\) is concave, then \(\sum w_i f(x_i) \leq f(\sum w_i x_i)\), where \(w_i\) denote respective weights such that \(\sum w_i = 1\). Since \(f(x) = \ln(x)\) is (strictly) concave on \((0, \infty)\), we have \(\sum w_i \ln(x_i) < \ln(\sum w_i x_i)\). Taking the exponential of both sides, we obtain

$$\Pi_{MX} < \sum w_i x_i.$$  

This completes the proof.
Given that $\sigma > 1$, for any level of productivity $\phi > 0$, we must have

$$\Pi_X(\phi) < \Pi_{MX}(\phi).$$

An implication of this inequality is that conditional on importing, the cut-off productivity for exporting is lower. In other words, a firm that uses high-quality imported intermediates from the North has a greater chance of exporting to the North.

**Result 7** *Conditional on importing intermediates, the probability of exporting is higher.*

Given Result 7, a fall in the cut-off productivity for importing would lead to an increase in the probability of exporting to the North. Due to the parameter $z$ the effect of importing on the probability of exporting can be industry-specific: in industries with a low value of $z$, the elasticity of product quality with respect to input quality is high. According to Result 6, the cut-off productivity for importing high-quality imported intermediates from the North is lower in industries with a high elasticity of product quality with respect to input quality (low $z$). Thus input trade liberalisation would benefit such industries to a greater extent. This prediction is in line with the empirical findings reported by Bas (2012). She uses plant-level Argentine data and finds that the effect of input trade liberalisation on firm performance varies across industries.

## 4 Conclusion

Recent transaction-level datasets from Southern countries have unearthed new findings that are either missing in, or conflicting with the predictions of existing international trade models. First, I discuss that they all come together nicely into a consistent story about a firm’s integration into global markets, which can be titled *within-firm differentiation*—a strategy that consists of two elements: quality-to-market and sourcing intermediates from multiple sources.

In the model, an increase in the variety of intermediates imported by a Southern firm from the North leads to quality upgrading. The variety of imported intermediates increases with specific trade costs on final goods, but decreases with ad valorem trade costs on final goods and on intermediates. There is also a
second-derivative effect: the incentive to upgrade product quality is increasing in consumer demand for quality and the ratio of specific to ad valorem trade costs on final goods. Another result derived from the model suggests that more productive Southern firms use more variety of high-quality imported intermediates. The result holds even in the absence of any complementarity between firm productivity and input quality, provided there are specific trade costs on final goods.

I also embed the model in the heterogeneous firms trade model introduced by Chaney (2008) to study the decision to import and export. The extension generates the Linder’s hypothesis (Linder 1961): firms located in markets where consumer demand for quality is high are better at producing high-quality products. Also, I derive the following two results. First, even in the absence of fixed importing cost, there exists a cut-off productivity for Southern firms to import intermediates from the North. Second, input trade liberalisation is more likely to increase the probability of exporting in industries where intermediate quality is an important determinant of product quality.

The model presented in the paper is motivated by a number of recent empirical findings. The model is able to explain these findings, and it also generates new predictions. The next task is to confront these predictions with data.

A Appendix

A.1 Profit maximisation problem

The maximisation problem is subject to two inequality constraints: (i) \( A_N > 0 \), (ii) \( A - A_N > 0 \). We can formulate the Langrangian as follows:

\[
\mathcal{L}(\phi) = \frac{\mu Y}{P} (q(\phi, \Psi)^\gamma)^{\sigma-1} \left( \frac{P^{ci}(\phi)}{P} \right)^{-\sigma} \left\{ p - \phi^{-\alpha} \left[ r_S A + (\tau_N - r_S)A_N \right] \right\} + y_1 A_N + y_2 (A - A_N),
\]

where \( y_1 \) and \( y_2 \) are the corresponding Lagrange multipliers. Note that (i) and (ii) cannot both bind.
The respective first-order conditions with respect to \( p \) and \( A_N \) are

\[
Bq^\gamma(\sigma^{-1})(p^{cij})^{-\gamma(\sigma+1)}\left[-\sigma \tau_o (p - C) + \tau_o p + t\right] = 0
\]

\[
Bq^\gamma(\sigma^{-1}) - \sigma \left[ \gamma(\sigma - 1) (p - C) \frac{\partial q}{\partial A_N} - q \frac{\partial C}{\partial A_N} \right] + y_1 - y_2 = 0,
\]

where \( B = \mu Y P \sigma^{-1} \). Also, the following Complementary Slackness conditions should hold:

\[
y_1 A_N = 0; \quad y_1 \geq 0,
\]

\[
y_2 (A - A_N) = 0; \quad y_2 \geq 0.
\]

To check the second-order conditions, define the corresponding Hessian matrix:

\[
H = \begin{bmatrix}
\frac{\partial^2 \Pi}{\partial p^2} & \frac{\partial^2 \Pi}{\partial p \partial A_N} \\
\frac{\partial^2 \Pi}{\partial A_N \partial p} & \frac{\partial^2 \Pi}{\partial A_N^2}
\end{bmatrix}.
\]

If \( H \) is negative definite at an interior critical point, then the profit function is strictly concave at that point. In that case, since constraints (i) and (ii) are concave, then the point is a unique maximum. \( H \) is negative definite if the following conditions hold:

\[
\frac{\partial^2 \Pi}{\partial p^2} < 0, \quad \frac{\partial^2 \Pi}{\partial A_N^2} < 0, \quad |H| > 0.
\]

At any interior solution for \( A_N \), we have

\[
\frac{\partial^2 \Pi}{\partial p^2} = -Bq^\gamma(\sigma^{-1})(p^{cij})^{-\gamma(\sigma+1)} \tau_o (\sigma - 1) < 0,
\]

\[
\frac{\partial^2 \Pi}{\partial A_N^2} = Bq^\gamma(\sigma^{-1}) - \sigma \left[ \gamma(\sigma - 1) (p - C) \frac{\partial q}{\partial A_N} - q \frac{\partial C}{\partial A_N} \right] - (1 + \gamma(\sigma - 1)) \frac{\partial q}{\partial A_N} \frac{\partial C}{\partial A_N} < 0
\]

\[
\frac{\partial^2 \Pi}{\partial A_N \partial p} = Bq^\gamma(\sigma^{-1})(p^{cij})^{-\gamma(\sigma+1)} \sigma \tau_o \frac{\partial C}{\partial A_N} > 0.
\]

As \( \frac{\partial^2 \Pi}{\partial p^2} < 0 \) and \( \frac{\partial^2 \Pi}{\partial A_N^2} < 0 \), the first two conditions in (27) are satis-
Thus, if $\Phi$ is given by

$$A = \text{the solution to (25) and (26) setting } \sigma = \frac{\partial q}{\partial A_N^{N}} \Omega$$

where $\Phi = B^2 q^{2\gamma(\sigma-1)-1} (p^{\text{eff}})^{-2(\sigma+1)} \tau_0$. Using, first, (25) to derive $p^{\text{eff}} = \sigma - 1 (\tau_0 C + t)$, and then (26), we obtain

$$|H| = \left( \frac{\partial A_N^{N}}{\partial N} \frac{\partial \Pi}{\partial \Pi} - \left( \frac{\partial^2 \Pi}{\partial A_N^{N} \partial p} \right)^2 \right)$$

$$= - \left\{ \begin{array}{l}
\frac{\partial^2 q(\phi, \Psi)}{\partial A_N^{N} \partial q} - (1 + \gamma(\sigma - 1)) \left( \frac{\partial q}{\partial A_N^{N}} \right) \right. \left. + q\sigma \left( \frac{\partial q}{\partial A_N^{N}} \right) \right\}
\frac{\partial C}{\partial A_N^{N}} \frac{\partial q}{\partial A_N^{N}}
\right\}

< 0 \text{ since } \gamma \in (0, 1)

Thus, if $A_N^{N} \in (0, A)$, then the pair $(p^*, A_N^{N})$ is a unique maximum. The pair is the solution to (25) and (26) setting $y_1 = y_2 = 0$.

**A.2 Comparative statics with respect to preferences and trade costs**

The comparative statics for $A_N^{N}$ with respect to the cost of importing intermediates is given by

$$\frac{\partial A_N^{N}}{\partial \Pi} = -\Pi_{A_N} \Pi_{P_P} + \Pi_P \Pi_{A_N P} \frac{H}{|H|}.$$
Using the relevant expressions from subsection 2.2 and Appendix A.1, we obtain
\[ \frac{\partial A^*_N}{\partial \tau} = \frac{B^2 \left( \frac{p^{cif}}{q^f} \right)^{-2(\sigma+1)} q^{-1} \tau_o \partial C \left\{ -p^{cif} (\sigma - 1) \left[ \gamma (\sigma - 1) \frac{\partial q}{\partial A_N} + \frac{q}{A_N} \right] + q \sigma^2 \tau_o \frac{\partial C}{\partial A_N} \right\} - B^2 \left( \frac{p^{cif}}{q^f} \right)^{-2(\sigma+1)} q^{-1} \tau_o \frac{C}{A_N} \partial C \left( \frac{q^{cif} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2}}{\gamma \frac{\partial A_N}{\partial A_N}} + q \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] \right\} \].

Simplifying and using (26), we obtain
\[ \frac{\partial A^*_N}{\partial \tau} = \frac{\sigma \frac{\partial C}{\partial A_N} \left\{ 1 - \frac{C + \frac{\tau_o}{\tau_{rn}}}{A_N \phi^{1/\gamma (\sigma - 1) + \sigma}} \right\} - \left\{ \frac{q^2 \frac{\partial C}{\partial A_N} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2}}{\gamma^2 \frac{\partial A_N}{\partial A_N}} + q \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] \right\} \] \[ = - \frac{\sigma \frac{\partial C}{\partial A_N} \phi^{-\alpha r_s A + \frac{\gamma}{\tau_{rn} - r_s}} \left\{ \frac{q \frac{\partial C}{\partial A_N} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2}}{\gamma^2 \frac{\partial A_N}{\partial A_N}} + q \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] \right\} \] \[ < 0, \]

as \( \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2} < 0 \) and \( \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] < 0 \) given \( \gamma \in (0, 1) \). The numerator does not depend on \( \gamma \), while the denominator is decreasing in \( \gamma \):

\[ - \frac{\partial}{\partial \gamma} \left\{ \frac{q \frac{\partial C}{\partial A_N} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2}}{\gamma^2 \frac{\partial A_N}{\partial A_N}} + q \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \left[ -\frac{1}{\gamma} (1 + \gamma (\sigma - 1)) + \sigma \right] \right\} \] \[ = 1 \frac{\gamma^2}{\gamma^2} \left\{ \frac{q \frac{\partial C}{\partial A_N} \frac{\partial^2 q(\phi, \Psi)}{\partial A_N^2}}{\gamma \frac{\partial A_N}{\partial A_N}} - q \frac{\partial C}{\partial A_N} \frac{\partial q}{\partial A_N} \right\} < 0. \]

So, \( \frac{\partial^2 A^*_N}{\partial \gamma \partial \tau} < 0 \). Similarly, \( \frac{1}{\tau_o} \) is present only in the numerator, and it is easy to see
\[ \frac{\partial^2 A^*_N}{\partial (1/\tau_o) \partial \tau} < 0. \]

This completes the proof.
A.3 Derivation of the cut-off productivity in quasi-autarky

Variable profits of a Southern firm with productivity $\phi$ are equal to

$$\Pi^{var}(\phi) = \frac{\mu}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} Y_S P_S^{\sigma-1} \left( \frac{\phi - \frac{\alpha}{\sigma} \lambda r_N}{\phi \gamma_S^z \lambda \gamma_S(1 - z)} \right)^{1-\sigma}.$$  

For the least productive firm, variable profits only cover the fixed cost:

$$\frac{\mu}{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} Y_S P_S^{\sigma-1} \left( \frac{\phi - \frac{\alpha}{\sigma} \lambda r_N}{\phi \gamma_S^z \lambda \gamma_S(1 - z)} \right)^{1-\sigma} - f = 0$$

$$\implies \phi^{(\sigma - 1)(\alpha + \gamma_S z)} = \frac{f}{Y_S} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma - 1} \left( \frac{\lambda r_N}{\lambda \gamma_S(1 - z)} \right)^{\sigma - 1}.$$  

(28)

The price index is given by

$$P_S^{1-\sigma} = r_S L_S \left( \frac{\lambda r_N}{\lambda \gamma_S(1 - z)} \right)^{1-\sigma} \int_{\phi^*}^{\phi} (\sigma - 1)(\alpha + \gamma_S z) dG(\phi)$$

$$\implies P_S^{1-\sigma} = r_S L_S \left( \frac{\sigma \lambda r_N}{(\sigma - 1) \lambda \gamma_S(1 - z)} \right)^{1-\sigma} \int_{\phi^*}^{\phi} (\sigma - 1)(\alpha + \gamma_S z) dG(\phi)$$

$$= r_S L_S \left( \frac{\sigma \lambda r_N}{(\sigma - 1) \lambda \gamma_S(1 - z)} \right)^{1-\sigma} \frac{\nu}{\nu - (\sigma - 1)(\alpha + \gamma_S z)} \phi^{(\sigma - 1)(\alpha + \gamma_S z) - \nu}. $$

Substitute this into the expression in (28) to obtain:

$$\phi^\nu = \frac{\sigma}{\mu} \frac{\nu}{\nu - (\sigma - 1)(\alpha + \gamma_S z)} \frac{f}{1 + \pi},$$

where $\pi$ denotes dividend per share.

The marginal cost of production and product quality of a Southern firm with productivity $\phi$ are given by

$$C(\phi) = \phi^{-\alpha} r_N, $$

$$q(\phi) = \phi^z.$$ 

Then, denoting the least productive active firm in the North by $\phi^*_N$, we can derive
the price index in the North as:

\[ P_N^{1-\sigma} = r_N L_N^1 r_N^{1-\sigma} \int_{\phi_N}^{\phi_N} \phi \left( \sigma^{-1} (\alpha + \gamma_N z) \right) dG(\phi) \]

\[ \Rightarrow P_N^{1-\sigma} = r_N L_N^1 r_N^{1-\sigma} \int_{\phi_N}^{\phi_N} \phi \left( \sigma^{-1} (\alpha + \gamma_N z) \right) dG(\phi) \]

\[ = r_N L_N^1 r_N^{1-\sigma} \frac{\sigma^{\nu} (\sigma - 1)(\alpha + \gamma_N z)}{v - (\sigma - 1)(\alpha + \gamma_N z)} \phi_N^{(\sigma-1)(\alpha+\gamma_N z)-\nu}. \]

Using these, we can show that the following holds for the least productive active firm in the Northern market:

\[ \phi_N^{\nu} = \frac{\sigma}{\mu} \frac{\nu}{v - (\sigma - 1)(\alpha + \gamma_N z)} \frac{f}{1 + \pi}. \]

References


